Multi-Context Models for Reasoning under Partial Knowledge: Generative Process and Inference Grammar

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Motivation

Complete probabilistic knowledge of a domain: Not always available

Settings often arise for which
- an individual merely possesses partial knowledge, yet,
- is expected to give adequate answers to a variety of posed queries.

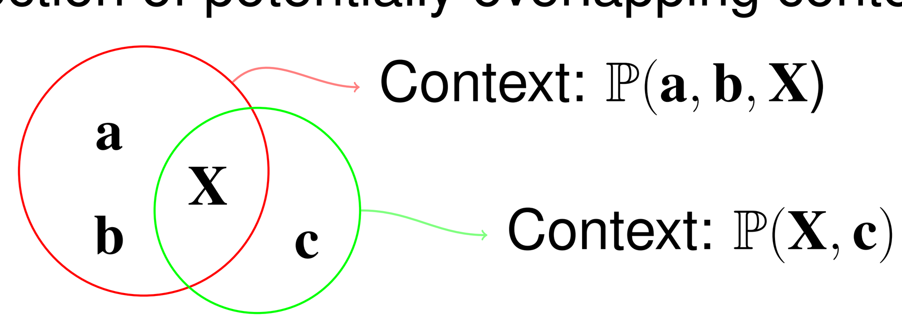
Our proposal: Multi-Context Model (MCM)

MCM
- **is** a graphical model to represent the state of partial knowledge regarding a domain;
- **forms** a middle ground between Probabilistic Logic, Bayesian Logic, Probabilistic Graphical Models.

What is a Context?

Stated elegantly by Pearl: *"this state of partial knowledge is more common, because we often begin thinking about a problem through **isolated frames**, paying no attention to inter-dependencies."—[emphasis added.]*

MCM encodes the probabilistic knowledge of a domain as a collection of potentially overlapping contexts.



Generative Process

Key Question: How to generate a collection of contexts:
- Probabilistically consistent
- Gradually over time
- Freely

When generating a new context, all its parts induced by already present contexts have to be, altogether, contained within some already present context.

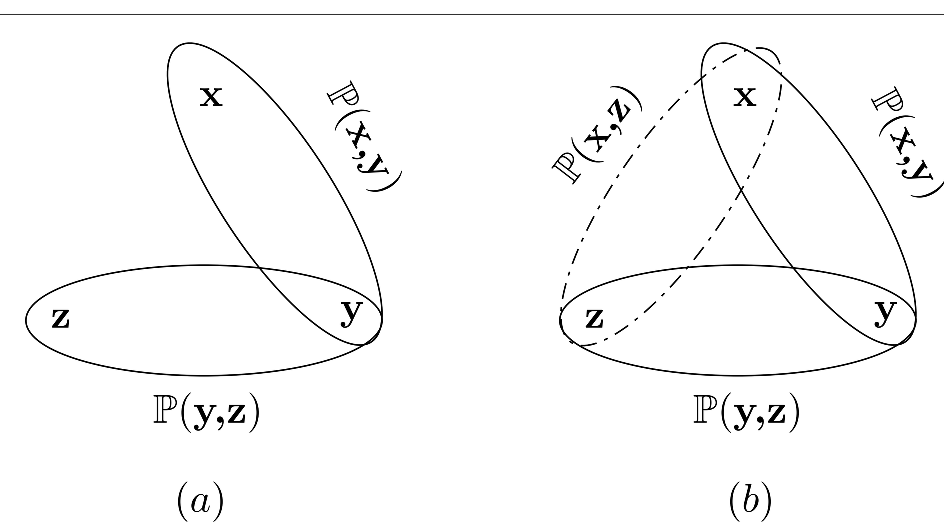
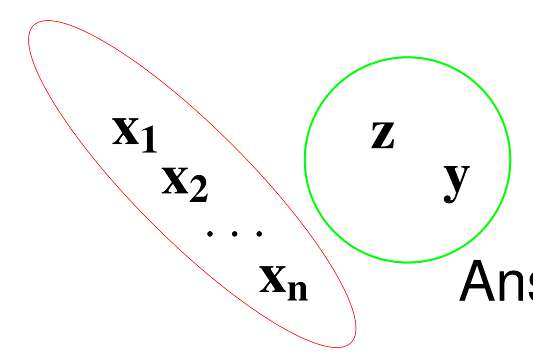


Figure: The dash-dotted contexts cannot be freely assigned.

Inference in MCMs

Objective: Evaluate $\mathbb{P}(\mathbf{O} = O|\mathbf{E} = E)$.

Motivating Example



Query: $\mathbb{P}(y|x_i) = ?$

Ans: $\mathbb{P}(y|x_i)_\downarrow = \left[\frac{\mathbb{P}(y)-\mathbb{P}(\bar{x}_i)}{\mathbb{P}(x_i)}\right]^+$

Conventional Approach Write down *all* the available information as a list of linear equations and solve it as a Linear Program (LP)

Drawback Inability to distinguish between the relevant information from the irrelevant for the posed query

Proposal

Distinguish between relevant and irrelevant information and dismiss the latter.

High Level Reasoning: Identify relevant quantities (Inference Grammar)

↓

Low-Level Reasoning: (Intra-contextual Inference)

↓

Derive (optimum) bounds to the posed query: (State and solve as an LP)

Intra-Contextual Inference Problems

Variables in **E** and **O** all belong to the same context.

Rationale: Take advantage of the rich independence structure potentially governing the context to accomplish the task of inference in a computationally efficient way.

Inter-Contextual Inference Problems

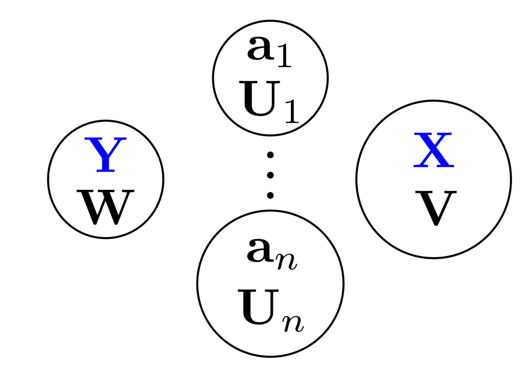
Variables in **E** and **O** do not belong to the same context.

For more information...

See our ArXiv paper

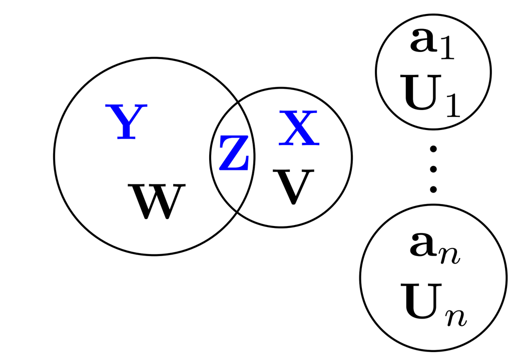
Sample Inter-Contextual Inference Rules

(a)



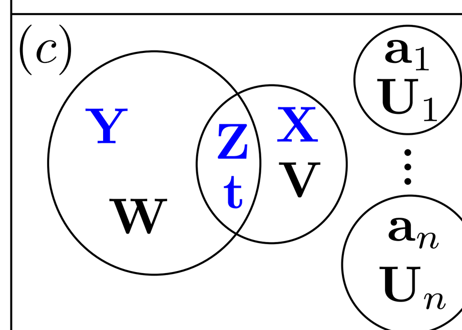
$\mathbb{P}(X|Y)_\downarrow = [\frac{\mathbb{P}(X)-\mathbb{P}(\bar{Y})}{\mathbb{P}(Y)}]^+$

(b)



$\mathbb{P}(X|Y,Z)_\downarrow = [\frac{\mathbb{P}(X|Z)-\mathbb{P}(\bar{Y}|Z)}{\mathbb{P}(Y|Z)}]^+$

(c)



$\mathbb{P}(X|Y,Z,t)_\downarrow = [\frac{\mathbb{P}(X|Z,t)-\mathbb{P}(\bar{Y}|Z,t)}{\mathbb{P}(Y|Z,t)}]^+$

$\mathbb{P}(X|Y,Z)_\downarrow = \sum_{t\in Val(\mathbf{t})} \mathbb{P}(X|Y,Z,\mathbf{t}=t)_\downarrow \, \mathbb{P}(\mathbf{t}=t|Y,Z)$

Two Key Properties of the Rules:
- Scale-Invariance.
- Variables are mere place-holders (as in Predicate Logic).

MCM Rules hint towards an algorithm (*inference grammar*), $\mathcal{I}^*$.

Inter-Contextual Inference Problems: Nestedness and Transformation

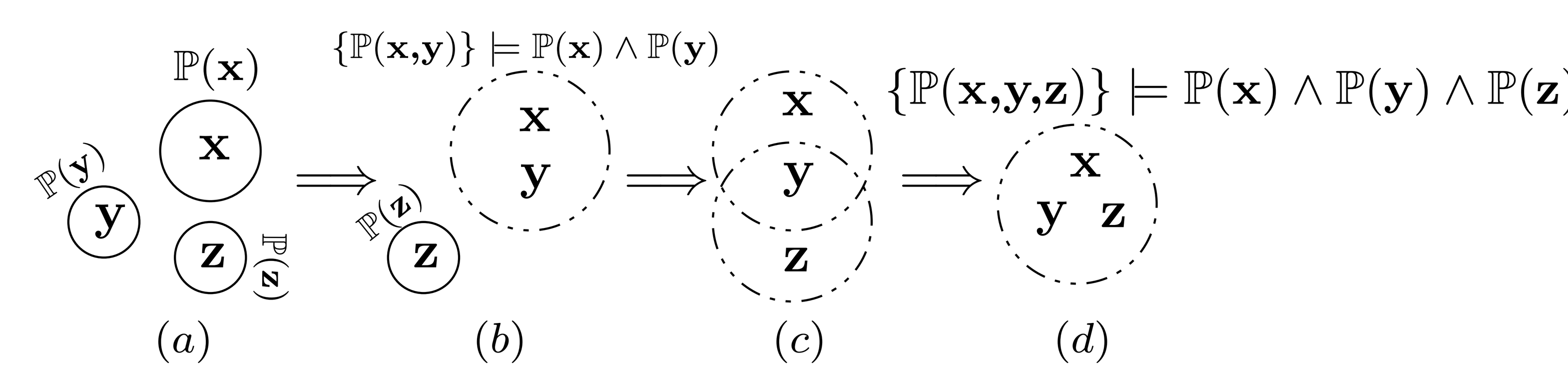


Figure: Transformation and Hierarchical Construct.

Example:

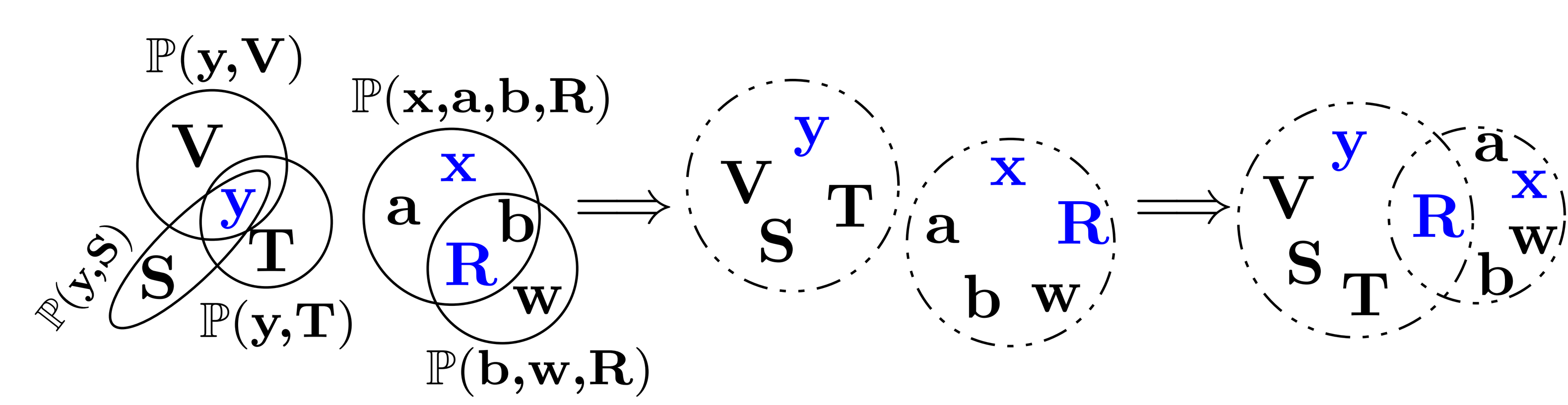


Figure: Transformation: Sample Case.

(Right-Most): $\mathbb{P}(x|y,R)_\downarrow = [\frac{\mathbb{P}(x|R)+\mathbb{P}(y|R)-1}{\mathbb{P}(y|R)}]^+,$

(Middle): $\mathbb{P}(y|R)_\downarrow = [\frac{\mathbb{P}(y)-\mathbb{P}(\bar{R})}{\mathbb{P}(y)}],$

$\therefore \mathbb{P}(x|y,R)_\downarrow = [\frac{\mathbb{P}(x|R)+\mathbb{P}(y|R)_\downarrow-1}{\mathbb{P}(y|R)_\downarrow}]^+.$

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