Supplementary Material for
IN-NETWORK LINEAR REGRESSION WITH ARBITRARILY SPLIT DATA MATRICES†

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Proof of Proposition 2

We shall prove the following result:

Proposition. *Consider a real vector space $\mathcal{V}$, and let $v_1, \dots, v_m$ be vectors in $\mathcal{V}$. Suppose that $g\colon \mathcal{V} \to \mathbb{R}$ is a convex function, and let ϵ be a real number. If*

$$g(v_1 + \cdots + v_m) \le \epsilon, \tag{A}$$

then for all positive numbers $p_1, \dots, p_m$ that sum to 1, there exist vectors $\Delta v_1, \dots, \Delta v_m$ in $\mathcal{V}$ that sum to zero such that

$$g\big(p_1^{-1}(v_1 + \Delta v_1)\big) \le \epsilon, \quad \dots, \quad g\big(p_m^{-1}(v_m + \Delta v_m)\big) \le \epsilon. \tag{B}$$

If, on the other hand, (B) *holds for some positive numbers $p_1, \dots, p_m$ that sum to 1 and some vectors $\Delta v_1, \dots, \Delta v_m$ in $\mathcal{V}$ that sum to zero, then* (A) *holds.*

Proof. First, suppose that (A) holds. For any positive numbers $p_1, \dots, p_m$ that sum to 1, define

$$\Delta v_i = p_i(v_1 + \cdots + v_m) - v_i, \qquad i = 1, \dots, m.$$

Then, not only is it true that $\Delta v_1 + \cdots + \Delta v_m = 0$, but since

$$g\big(p_i^{-1}(v_i + \Delta v_i)\big) = g\big(p_i^{-1}(v_i + p_i(v_1 + \cdots + v_m) - v_i)\big) = g(v_1 + \cdots + v_m) \le \epsilon$$

for every i, it follows that (B) holds. Since the positive numbers $p_1, \dots, p_m$ that sum to 1 are arbitrary, (B) holds for all such numbers with the corresponding vectors $\Delta v_1, \dots, \Delta v_m$ that sum to zero.

Next, suppose that (B) holds for some positive numbers $p_1, \dots, p_m$ that sum to 1 and some vectors $\Delta v_1, \dots, \Delta v_m$ in $\mathcal{V}$ that sum to zero. Then, by Jensen's Inequality (due to the convexity of g),

$$g(v_1 + \cdots + v_m) = g\left(\sum_{i=1}^{m} p_i p_i^{-1}(v_i + \Delta v_i)\right) \le \sum_{i=1}^{m} p_i g\big(p_i^{-1}(v_i + \Delta v_i)\big) \le \epsilon,$$

and it follows that (A) holds. □

According to the result above, by implying (A), the assertion that

(B) *holds for* <u>*some*</u> *positive numbers $p_1, \dots, p_m$ that sum to 1 and some vectors $\Delta v_1, \dots, \Delta v_m$ in $\mathcal{V}$ that sum to zero*

implies that

for <u>*all*</u> *positive numbers $p_1, \dots, p_m$ that sum to 1, there exist vectors $\Delta v_1, \dots, \Delta v_m$ in $\mathcal{V}$ that sum to zero such that* (B) *holds.*

Since the converse clearly holds, these statements are, in fact, equivalent. This observation leads to Proposition 2.

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